

5. Capacitive Sensors

Sensor Technologies (James Cook University)

5. Capacitive Sensors

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Outline

- **Capacitors and Capacitance**
- Sensing Principles and Characteristics
- Sensor Design and Applications
- Electronic Interface Circuits

Capacitors and Capacitance

- A capacitor is a passive electrical or electronic component that can store energy in the form of an electric field.

Capacitance (in farad, F) \rightarrow $C = \frac{Q}{V}$ \leftarrow Charge (in coulomb, C)
 \leftarrow voltage difference between the two plates (in volts, V).

- V can be expressed in terms of the work done on a positive test charge q when it moves from the positive to the negative plate:

force (in newton, N) \rightarrow $V = \frac{Fd}{q} = Ed$ \leftarrow the distance between two plates
 \uparrow
electric field (in volt per meter, $V \cdot m^{-1}$ or newton per coulomb, $N \cdot C^{-1}$) between two parallel plates,

Capacitors and Capacitance

- Commonly used capacitors have a capacitance range from 1 pF to 1000 μ F.
- The voltage–current relationship of a capacitor is expressed by

$$V(t) = \frac{1}{C} \int I(t) dt$$

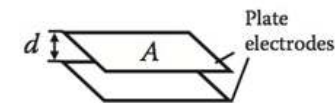
- Capacitors in series: $\frac{1}{C_{eq}} = \sum \frac{1}{C_i}$
- Capacitors in parallel: $C_{eq} = \sum C_i$

Capacitive Sensor

- Capacitive sensors are variable capacitors, that is, their capacitance changes during the sensing process.
- Capacitance is a function of geometry, dielectric constant, plate materials, and plate configuration.

- ✓ A is the area of the flat plate (in meter square, m^2),
- ✓ d is the spacing (in meter, m) between the plates;
- ✓ ϵ_0 is the electric permittivity in vacuum (8.8542×10^{-12} farad/m),
- ✓ ϵ_r is the relative dielectric permittivity (RDP, unitless) or dielectric constant of the media between the plates.
- ✓ r_1, r_2 , are the radii of the cylindrical plates (in meter, m),
- ✓ h is the length or height of the cylindrical plates (in meter, m).

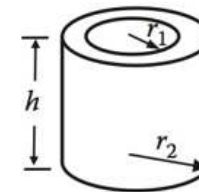
Parallel-(flat) plate capacitor



$$C = \frac{\epsilon_0 \epsilon_r A}{d}$$

Ratio A/d is called the *geometry factor* for a parallel-plate capacitor.

Cylindrical (coaxial) capacitor



$$C = \frac{2\pi\epsilon_0\epsilon_r h}{\ln(r_2/r_1)} \quad (h \gg r_2)$$

Ratio $2\pi h/\ln(r_2/r_1)$ is the *geometry factor* for a cylindrical capacitor.

Capacitive Sensor

- Dielectric constant ϵ_r is the ratio of the permittivity of a substance ϵ_p to the permittivity of free space ϵ_0 (i.e., $\epsilon_r = \epsilon_p/\epsilon_0$).
- It is an expression of the extent to which a material concentrates electric flux, or the ability of a material to store a charge under an applied electric field.
- ϵ_r has the range of 2 ~ 20 for most dry solid materials and often higher for liquids

RDP of Common Materials at Room Temperature

Material	RDP, ϵ_r	Material	RDP, ϵ_r	Material	RDP, ϵ_r
Air, vacuum	1	Paper	3.5	Mica	6
Clay	1.8–2.8	Silica glass	3.7	Rubber	7
Teflon	2	Silicon dioxide	3.9	Marble	8
Soft rubber	2.5	Nylon	4–5	Silicone	11–12
Wood	2.7	Porcelain	4.4	Alcohol	16–31
Silicone rubber	2.8	Diamond	5.5–10	Fresh water	80
Ice	3–4	Glass	5	Sea water	81–88

Capacitive Sensor

5.1 Example

The plates of a parallel capacitor have a separation of 2.85 mm, and each has an area of 10.2 cm². If a charge of 3.95×10^{-8} C is carried by each plate and the plates are in vacuum, find (1) the capacitance C , (2) the potential difference V between the plates, and (3) the magnitude of the electric field E between the plates.

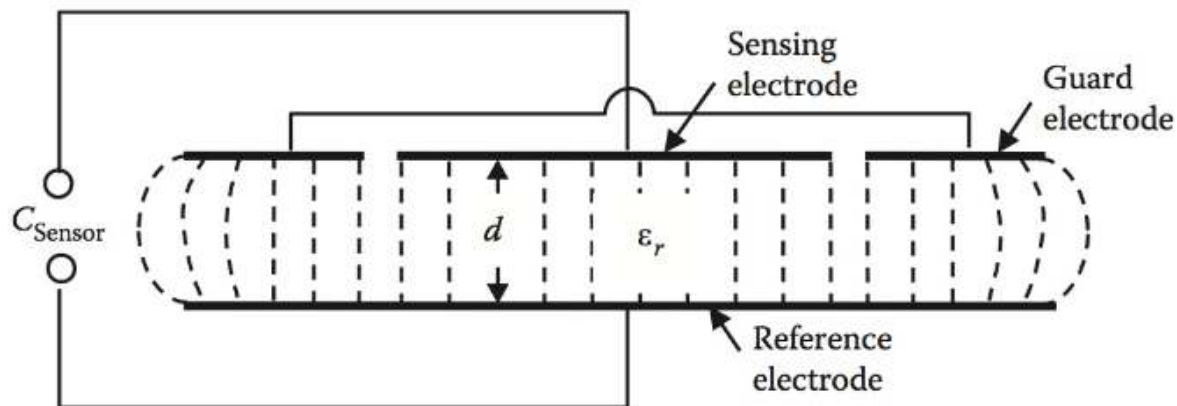
Outline

- Capacitors and Capacitance
- **Sensing Principles and Characteristics**
- Sensor Design and Applications
- Electronic Interface Circuits

Parallel-Plate Capacitive Sensors

- The simplest capacitive sensor consists of two parallel metal plates separated by a distance d
- To eliminate the fringe field or edge effect of the electrodes, the guard electrodes are added and kept at the same potential as the sensing electrode.
- The capacitance of a parallel plate capacitor can be found from

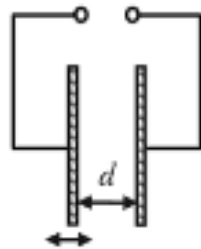
$$C = \frac{\epsilon_0 \epsilon_r A}{d}$$



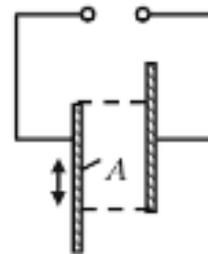
Parallel-Plate Capacitive Sensors

- Any variation in the dielectric constants ϵ_r , the plate area A , or the plate spacing d will cause a change in capacitance C .

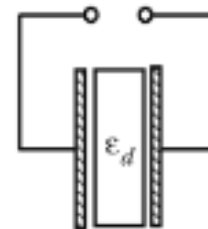
(a) Spacing variation



(b) Area variation



(c) Dielectric constant variation



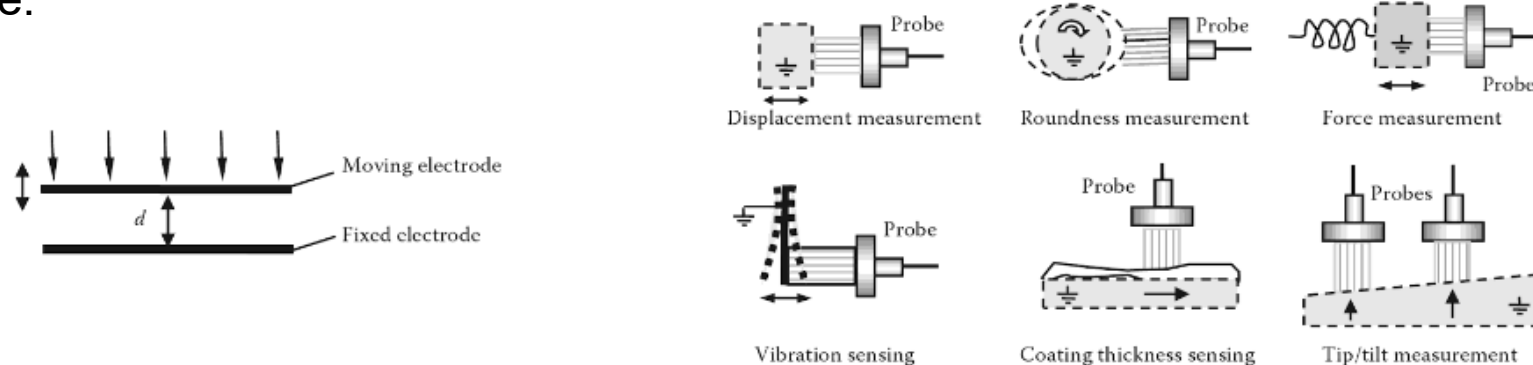
Spacing Variation Based Sensors

$$C = \frac{\epsilon_0 \epsilon_r A}{d}$$

- An inverse relationship exists between the spacing d and the capacitance C .
- This gives a large change in capacitance value with a small spacing change, but it also displays the nonlinearity.
- A bridge or signal conditioning circuit is often required to compensate for this nonlinearity.
- Compared to other capacitive sensors, spacing-variation-based sensors are generally more sensitive, but only suitable for a small displacement range (usually the spacing variation is less than the electrode size and in the range of micrometers).
- Area-based capacitive sensors, on the other hand, are less sensitive, but suitable for a larger displacement range.

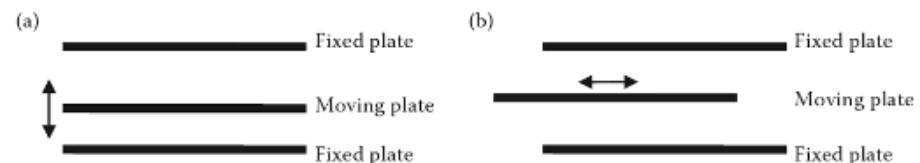
Spacing Variation Based Sensors

- Spacing-variation-based sensors can have single-plate, dual-plate, or multiplate designs.
- In the single-plate design, the sensing plate (or probe) functions as one electrode, while the target, made of a conductive material, functions as the second electrode.
- Traditionally, a capacitive sensor system has the probe driven and the target grounded.
- High-performance displacement sensors usually use small sensing surfaces and they are positioned close to the targets (0.25~2mm).
- In a dual-plate design, the sensor has two parallel-plate electrodes. When the moving plate experiences a force, pressure, or vibration, the distance d will change, causing a change in capacitance.



Spacing Variation Based Sensors

- Sometimes, temperature change can also affect d due to the thermal expansion of the plates and/or dielectric media. This can be compensated by applying the differential capacitance technique that uses three plates.
- By adding the third electrode, twin capacitors are formed: C_1 (between the top and middle plates) and C_2 (between the middle and bottom plates).
- Proper wiring and adding an amplifier circuit can make a sensor's output voltage proportional to $C_1 - C_2$, C_1/C_2 , or $(C_1 - C_2)/(C_1 + C_2)$ to compensate for temperature/humidity variation and small changes in dielectric constants.
- Both spacing-variation- and area-variation-based sensors have the three-plate configurations.



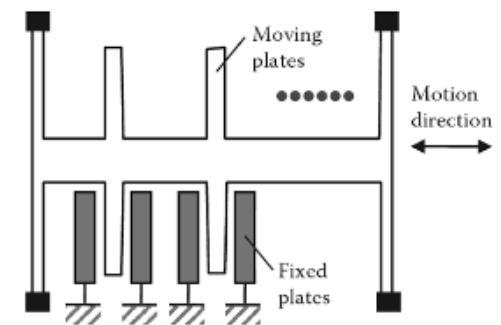
Spacing Variation Based Sensors

Example 5.2

Compare the capacitance values of (1) a two-plate capacitor with a spacing d , (2) a three-plate capacitor (in series) with a spacing $d/2$ between two adjacent plates, and (3) a three-plate capacitor (in parallel) with a spacing $d/2$ between two adjacent plates. Assume that the plate area A and dielectric constants κ maintain the same in each case.

Spacing Variation Based Sensors

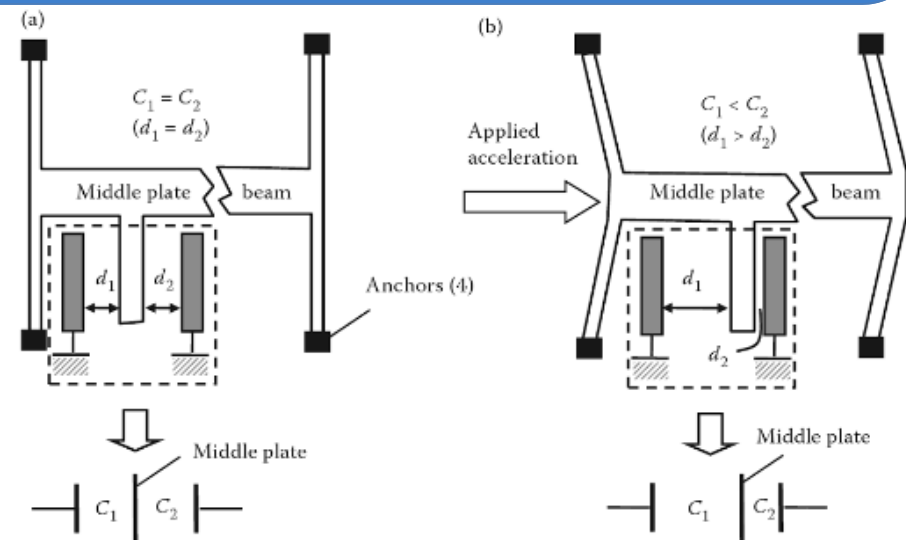
- To improve a sensor's measurement range, accuracy, sensitivity, stability, and signal-to-noise ratio, a multiplate configuration can be used, which could be a replication of the single-plate system many times or a replication of three-plate system many times.
- The figure shows a multiplate capacitive sensor that is often seen in a capacitive acceleration, vibration, and displacement measurements.



Spacing Variation Based Sensors

Example 5.3

The figures show a simplified version of a capacitive accelerometer at rest and under an applied acceleration, respectively. The actual sensor consists of 42 silicon finger cells (as fixed plates) and a common beam (as moving plates) together forming 42 groups of twin capacitors. The twin capacitors in each group, C_1 and C_2 , are serially connected. If no acceleration is applied, $C_1 = C_2$; if an acceleration is applied, $C_1 \neq C_2$. Assume that the sensor is under an acceleration causing $C_1 = 0.004\text{pF}$, $C_2 = 0.006\text{pF}$, what is the total capacitance of the entire sensor?



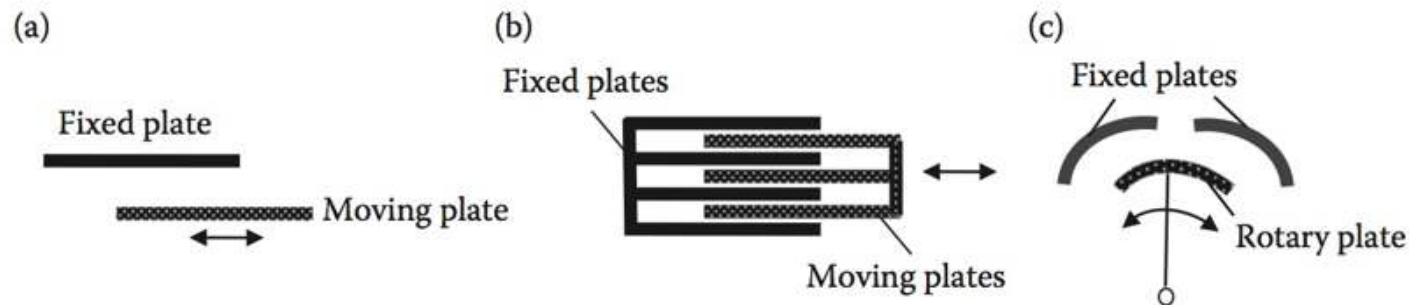
Area Variation Based Sensors

$$C = \frac{\epsilon_0 \epsilon_r A}{d}$$

- The capacitance of a flat plate capacitor is proportional to the area of the plate A .
- As one of the plates slides transversely, the overlapped area changes, causing the capacitance changes linearly.
- This change is usually converted into a voltage change.
- Often used for noncontact displacement measurement, especially when the movement is larger than the electrode dimension
- If the movement is less than the electrode size, the spacing-variation-based sensors are most suitable.

Area Variation Based Sensors

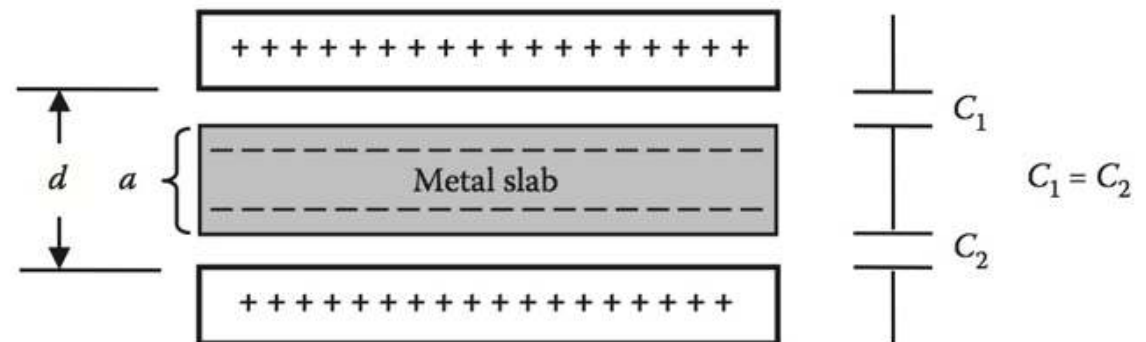
- Can be designed in one-pair or multi-pair plate configurations for either linear or rotary motion detection.
- A multi-plate structure improves a sensor's sensitivity and measurement accuracy, while requiring larger space and more expensive.



Area Variation Based Sensors

Example 5.4

An air capacitor consists of two flat plates, each with area A , separated by a distance d . Then a metal slab—with a thickness a ($< d$) and the same shape and size as the plates—is inserted between the two flat plates (at the middle) and parallel to the plates without touching either plate. Express the capacitance C of this three-plate capacitor in terms of capacitance C_0 when the metal slab is not inserted.



Dielectric Constant Variation Based Sensors

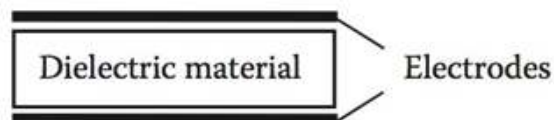
$$C = \frac{\epsilon_0 \epsilon_r A}{d}$$

- A linear relationship exists between dielectric constant ϵ_r and the capacitance C : the larger the ϵ_r , the bigger the C .
- Capacitance changes may depend on
 - the sensor's dielectric constant;
 - the target material's dielectric constant
- Dielectric-constant-variation-based capacitive sensors are widely used to detect
 - motion or proximity of nonconductive objects (e.g., dust, paper, plastics, and cloth),
 - chemical-substance
 - biocells.
- They can also measure RH, force, and pressure.

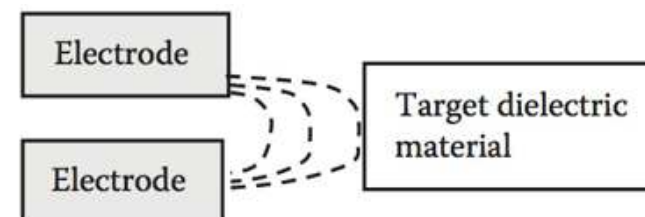
Dielectric Constant Variation Based Sensors

- Dielectric-based capacitive sensors have two basic designs.

- have two electrodes and one dielectric media in the sensor
- often seen in capacitive force, pressure, chemical, and humidity sensors
- the permittivity of the dielectric material changes when a sensor is subjected to a force, pressure, chemical gas, or water vapor.



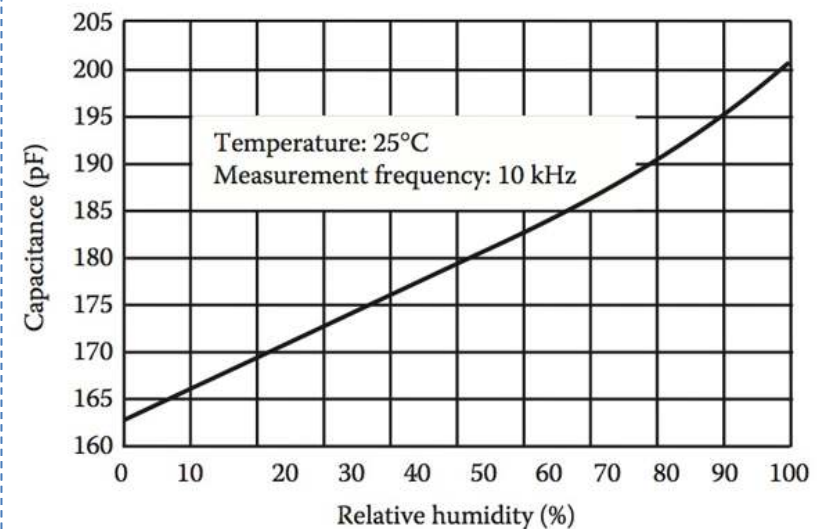
- have two electrodes but no dielectric media
- relies on the target's material to interfere with the electric field of the sensor to change capacitance
- The two metallic electrodes can be positioned like an "opened" capacitor
- The change in capacitance depends on the target material and the distance to the target.
- This type of sensor is often used to detect motion or proximity of nonconductive materials.



Dielectric Constant Variation Based Sensors

- Usually a change in a sensor's dielectric constant is due to
 - the dielectric material alters its density when a force or pressure is applied to the dielectric media
 - the dielectric material absorbs moisture
 - the dielectric media reacts with a target material.
- Capacitive humidity sensors, for example, require a dielectric material that easily absorbs and releases moisture based on the surrounding RH (relative humidity).
- The commonly used dielectric materials for humidity sensors are glass, ceramic, or silicone.

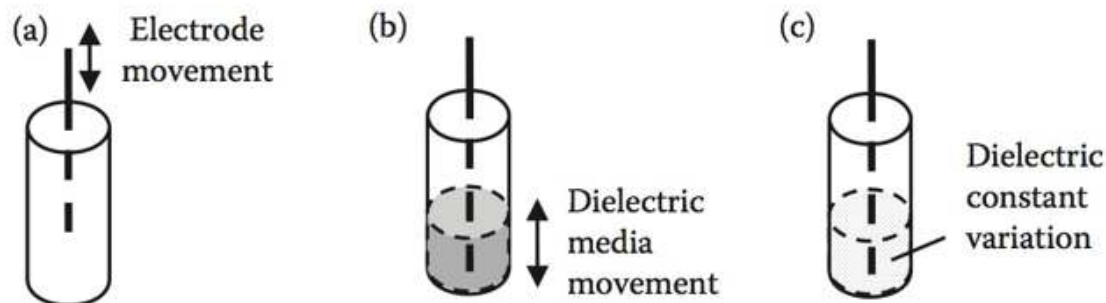
Typical response curve of an HS1100 capacitive humidity sensor.



Cylindrical Capacitive Sensors

- The three design principles of CCSs:
 - inner electrode movement,
 - dielectric media movement, and
 - dielectric constant variation.

$$C = \frac{2\pi\epsilon_0\epsilon_r h}{\ln(r_2/r_1)}$$



Electrode Movement Based Sensors

$$C = \frac{2\pi\epsilon_0\epsilon_r h}{\ln(r_2/r_1)}$$

- The moving electrode (usually the inner one) causes the overlap height h of the cylindrical plates to change, resulting in a change in capacitance C .
- A linear relationship exists between the overlap height h and the capacitance C : the bigger the height h , the larger the capacitance C .

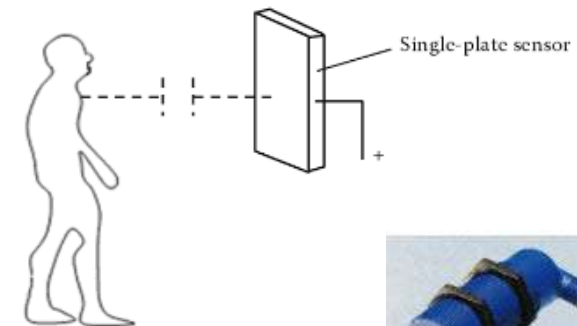
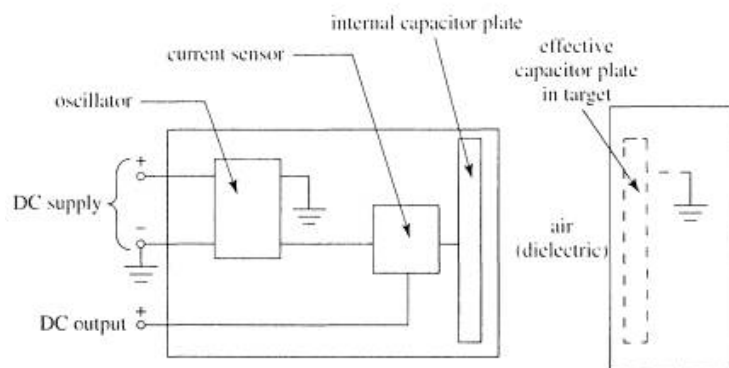
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Capacitive Proximity Sensor

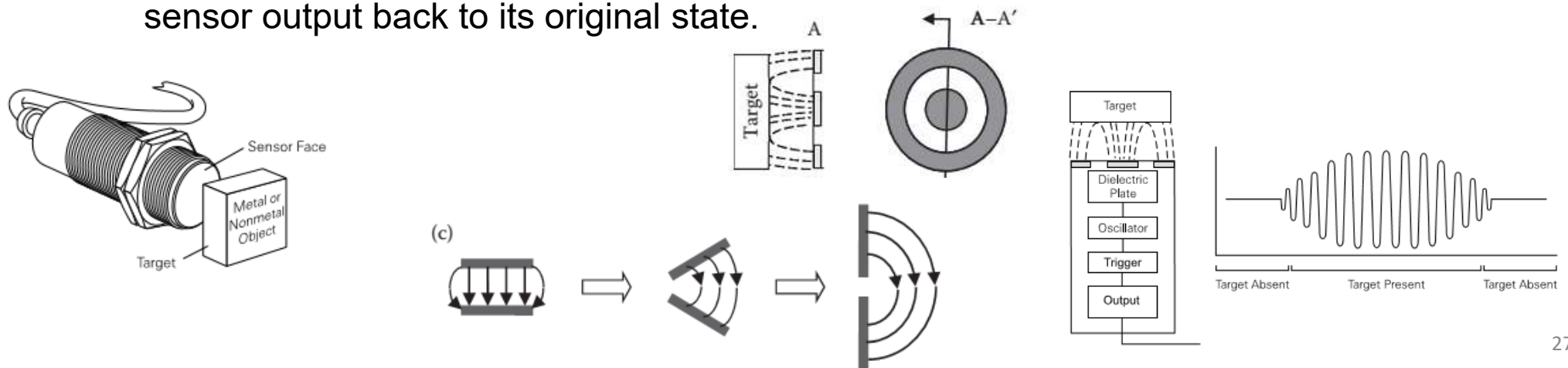
- Parallel Plate/Spacing Variation Based Sensors

- Capacitive proximity sensors based on spacing variation sense "target" objects due to the target's ability to be electrically charged.
- The figure shows a capacitive sensor formed by a human (a conductive target) and a single sensing plate. The capacitance increases when the human gets closer to the metal plate. By measuring this capacitance, the distance between the human and the sensing plate can be estimated.



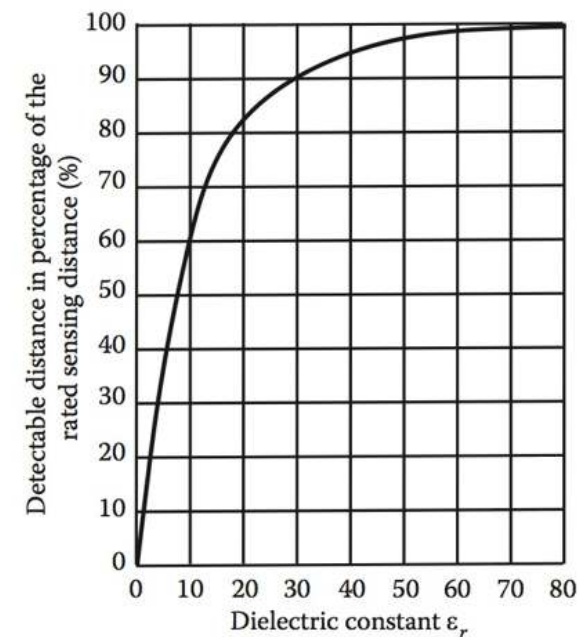
Capacitive Proximity Sensor

- Parallel Plate/Dielectric Constant Variation Based Sensors
 - Most capacitive proximity sensors are probe types with a typical internal layout.
 - The sensing surface is often formed by two concentric ring electrodes.
 - When an object nears the sensing surface and enters the electrostatic field of the electrodes, it alters the amount of electric flux reaching the second electrode and causes the capacitance to increase.
 - As the target moves away from the sensor, the capacitance decreases, switching the sensor output back to its original state.



Capacitive Proximity Sensor

- The larger the dielectric constant of the target material, the bigger the capacitance change, and the easier it is to detect the target.



Relationship between a target material's dielectric constant and the rated sensing distance for the Siemens proximity sensor.

Capacitive Proximity Sensor

Example 5.5

A Siemens capacitive sensor has a rated sensing distance of 9 mm and the target material is marble. What is the effective sensing distance?

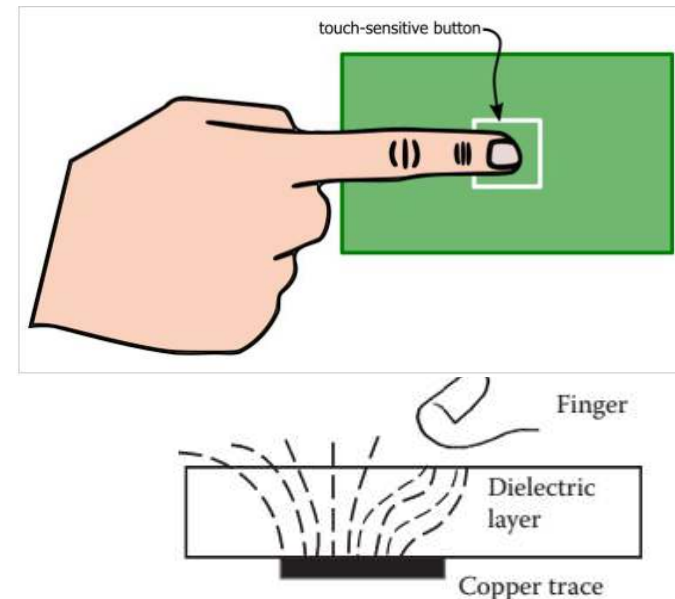
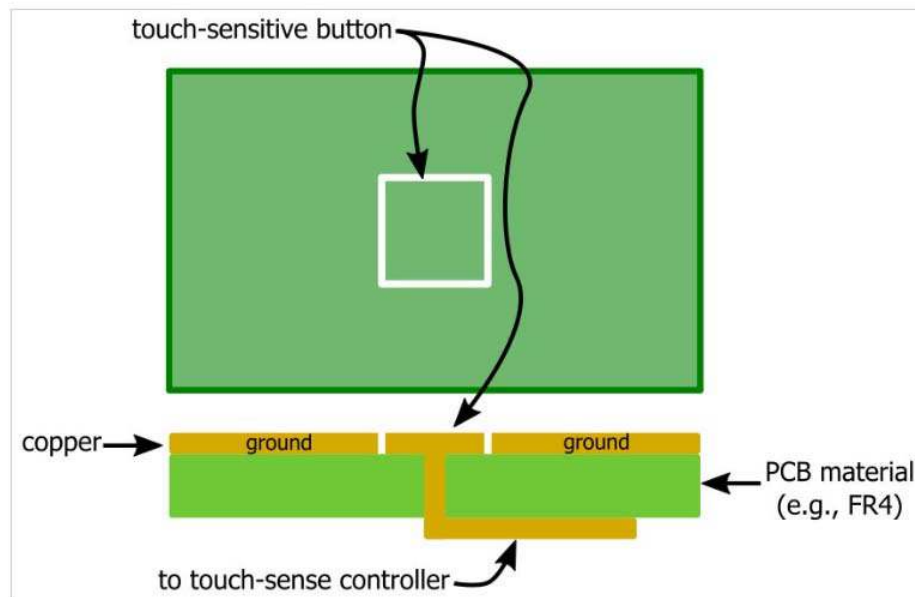
Capacitive Proximity Sensor

Example 5.6

A Siemens capacitive sensor has a normal (rated) sensing range S_n of 20 mm. Can this capacitive sensor be used to sense the presence of ammonia from behind a 3 mm glass panel? The dielectric constant for ammonia is 20, and for glass is 10.

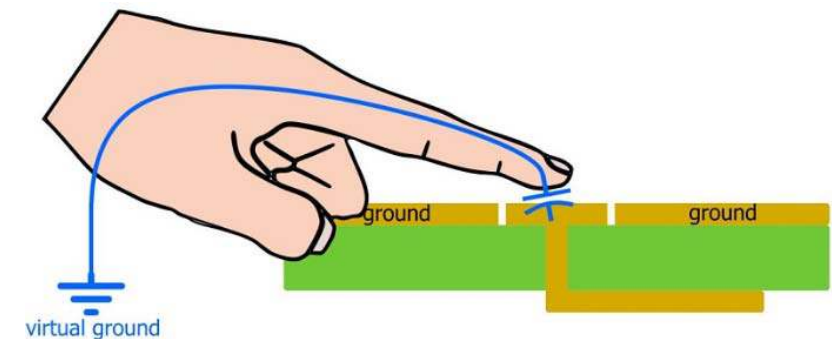
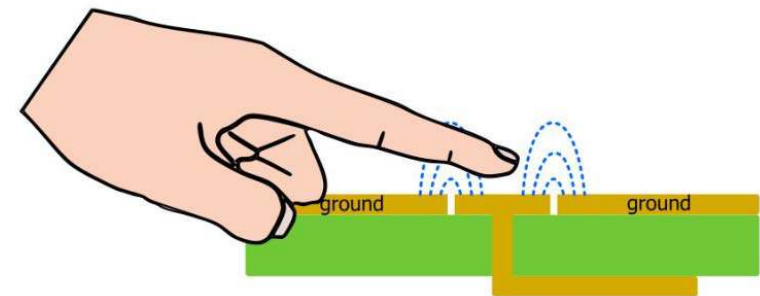
Capacitive Touch Sensor

- Parallel Plate/Spacing Variation Based Sensors, Dielectric Constant Variation Based Sensors



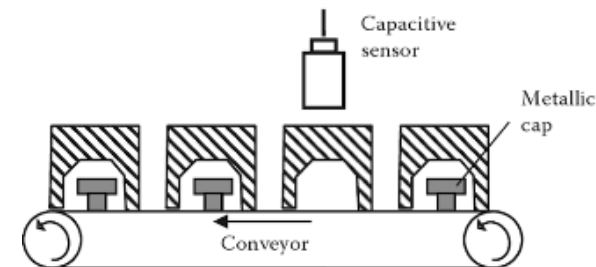
Capacitive Touch Sensor

- Finger as an dielectric
 - the finger's interaction with the capacitor's electric field represents an increase in the dielectric constant and hence an increase in the capacitance (similar to proximity sensor).
- Finger as a conductor
 - the finger becomes the second conductive plate of an additional capacitor (similar to proximity sensor).



Capacitive Defect Detection

- Parallel Plate/Spacing Variation Based Sensors
 - Because capacitive sensors have much higher sensitivities to conductors than non-conductors, they can be used to detect the presence or absence of metallic parts in completed assemblies.
 - As shown in the figure, if a cap is missing, the capacitive sensor can detect it.



Capacitive Pressure Sensor

- Over half of the sensors used in the chemical industry are capacitive pressure sensors.
- These sensors measure absolute, relative, and differential pressures, ranging from 1 psi (pounds per square inch) to 5000 psi.
- Accuracies of 0.1–0.5% of full scale are common for capacitive pressure sensors.
- Capacitive pressure sensor can be based on
 - Spacing Variation Based Sensors
 - Area Variation Based Sensors
 - Dielectric Constant Variation Based Sensors

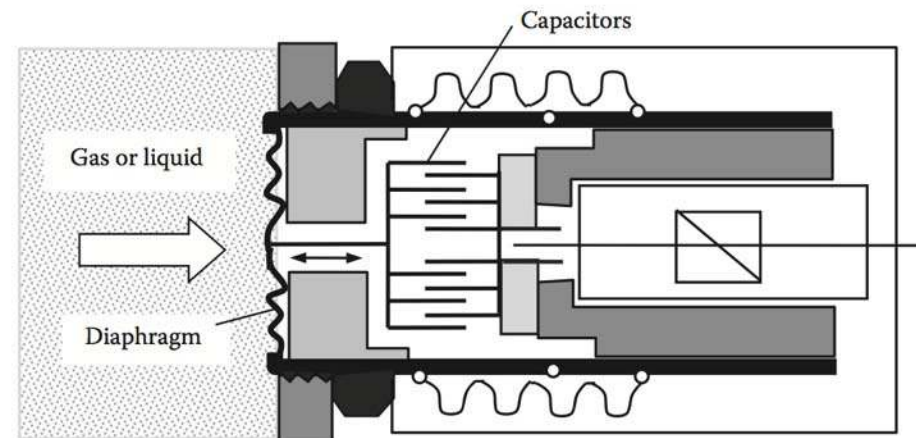
Capacitive Pressure Sensor

- Parallel Plate/Spacing Variation Based Sensors
 - consists of a fixed plate and a flexible plate (diaphragm). When liquid or gas enters the chamber, it presses the diaphragm, causing the spacing d and capacitance C to change. Thus, the capacitance variation directly relates to the pressure variation.

Capacitive Pressure Sensor

- Parallel Plate/Area Variation Based Sensors
 - consists of multiple plates and is more robust than other types of pressure sensors
 - One group of the electrodes is connected to the sensing diaphragm whose displacement is proportional to the pressure, causing the overlap area changes and therefore the capacitance change.
 - The unit forms a capacitor whose variation in plate area is determined by the movement of the diaphragm.

Capacitive pressure sensor with area variation. (Modified based on original drawing from VEGA Technique, France.)



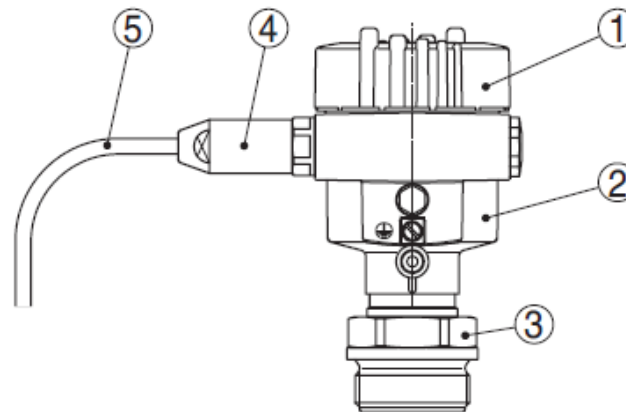
Capacitive Pressure Sensor

- VEGABAR 82/83

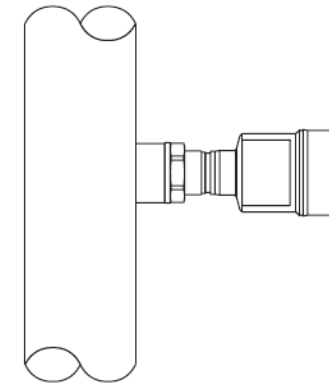
VEGABAR 82



VEGABAR 83



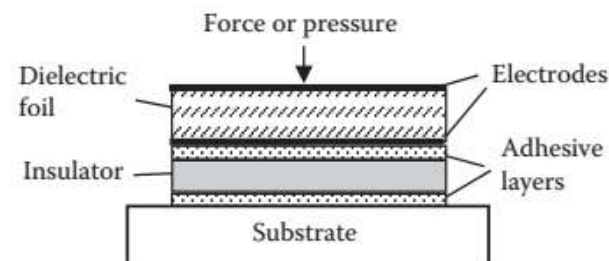
- 1 Housing lid with integrated display and adjustment module (optional)
- 2 Housing with electronics
- 3 Process fitting with measuring cell
- 4 Cable gland
- 5 Connection cable



The VEGABAR measures the pressure in a pipeline.

Capacitive Pressure Sensor

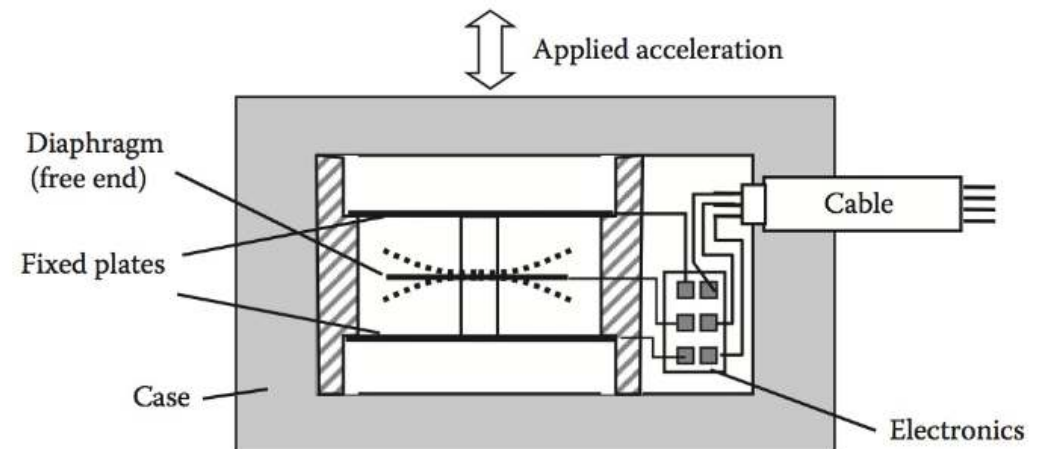
- Parallel Plate/Dielectric Constant Variation Based Sensors
 - the structure of a pellicular pressure sensor (about 80 μm thin) designed by ONERA French Aerospace Lab, France.
 - The sensor can detect the change in ϵ_r between two electrodes when a dynamic force or pressure is exerted on the plates.
 - The advantages of such pellicular sensors are their compactness, ability to detect micro pressure, resistance to vibration, and high bandwidth (e.g., 50 ~ 200 kHz).
 - The primary disadvantage is that they are temperature sensitive.



Capacitive Accelerometer

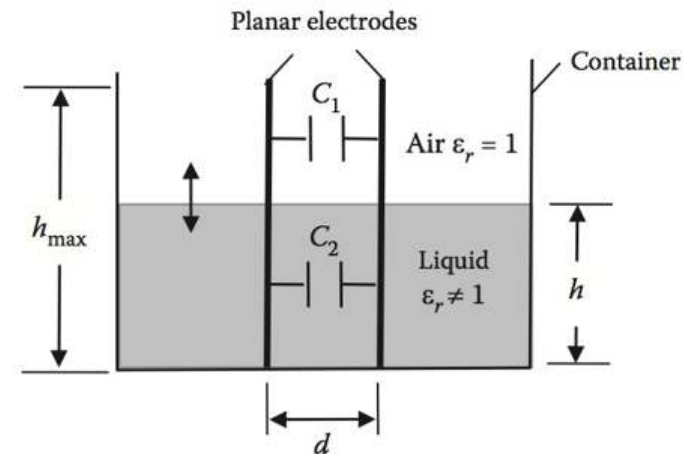
- Parallel Plate/Spacing Variation Based Sensors

- If one plate of a parallel capacitor moves subject to acceleration or vibration, a capacitive accelerometer or vibration sensor is formed.
- The figure shows a capacitive accelerometer which contains a diaphragm and two plates that sandwich the diaphragm
- The deflection causes a capacitance shift between the plates.
- Capacitive accelerometers generally have higher sensitivities than piezoelectric accelerometers.



Capacitive Level Sensor

- Parallel Plate/Dielectric Constant Variation Based Sensors
 - The capacitive level sensor evaluates the capacitance changes ($C - C_0$) due to the space between two planar electrodes being filled with a liquid, where C_0 is the capacitance of empty container,
 - The total capacitance C is the sum of the two parallelly connected capacitances C_1 and C_2 : $C = C_1 + C_2$.
 - ($C - C_0$) is approximately proportional to the filling height h .



Capacitive Level Sensor

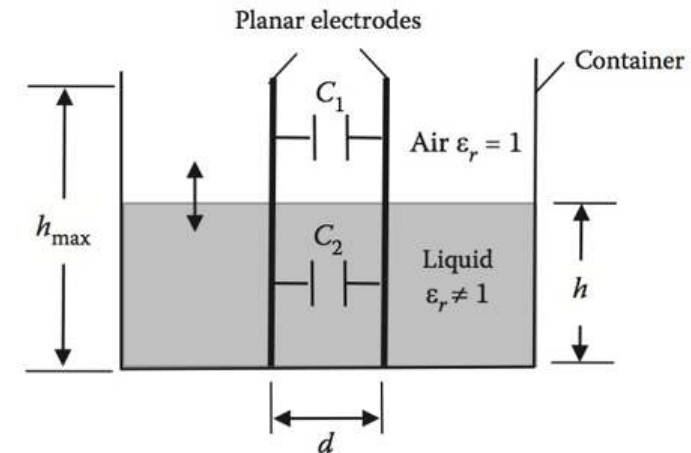
$$C_1 = \epsilon_0 b(h_{max} - h)/d \quad C_2 = \epsilon_0 \epsilon_r bh/d \quad C_0 = \epsilon_0 bh_{max}/d$$

Capacitance C depends on level h

$$C = C_0 + \epsilon_0(\epsilon_r - 1) \frac{bh}{d}$$

Level h can be expressed by

$$h = \frac{(C - C_0)d}{\epsilon_0(\epsilon_r - 1)b} = \frac{\Delta Cd}{\epsilon_0(\epsilon_r - 1)b}$$



Capacitive Level Sensor

- Cylindrical/Dielectric Constant Variation Based Sensors
 - The level of the dielectric media changes, causing the total capacitance C to change.
 - The two capacitors C_1 and C_2 are in a parallel connection.
 - Thus, the total capacitance C is

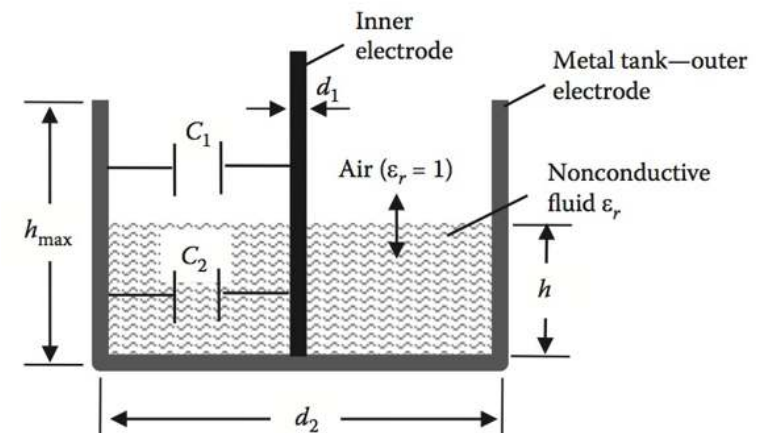
$$C = C_1 + C_2 = \frac{2\pi\epsilon_0(h_{max} - h)}{\ln(r_2/r_1)} + \frac{2\pi\epsilon_0\epsilon_r h}{\ln(r_2/r_1)}$$

$$= \frac{2\pi\epsilon_0}{\ln(r_2/r_1)} [h_{max} + (\epsilon_r - 1)h]$$

or $C = C_0 + \frac{2\pi\epsilon_0(\epsilon_r - 1)h}{\ln(r_2/r_1)}$, where $C_0 = \frac{2\pi\epsilon_0 h_{max}}{\ln(r_2/r_1)}$ when the tank is empty.

The level h of the dielectric material can be found by

$$h = \frac{(C - C_0) \ln(r_2/r_1)}{2\pi\epsilon_0(\epsilon_r - 1)}$$



Capacitive Level Sensor

- Capacitive Level Sensor developed by VEGA (France)
 - The probe and vessel walls are the capacitor plate.
 - Due to the higher dielectric constant of the product compared to air, the capacitance increases as the probe is gradually covered.

VEGACAP 62,63,64,66,69



Non-conductive liquids

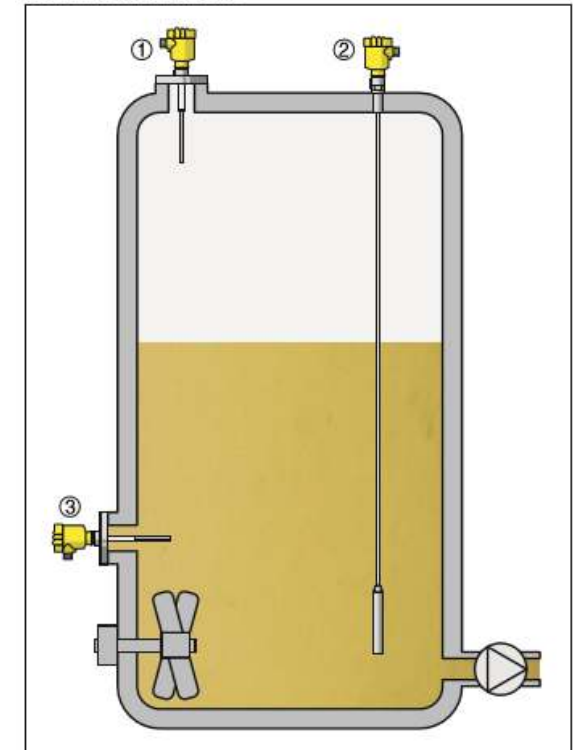


Fig. 2: Level detection in non-conductive liquids

- 1 VEGACAP 62 level switch for full signalling/overflow protection
- 2 VEGACAP 66 level switch for empty signalling/dry run protection
- 3 VEGACAP 62 level switch for level detection - laterally mounted

Capacitive Level Sensor

Example 5.7

The capacitive level sensor is chosen to measure a water level ($\epsilon_r = 80$) that can rise 300 mm high. If the capacitance reading is 503.4 pF, what is the water level? The width of the sensor's planar electrodes b is 20 mm, and the distance between the two plates d is 1 mm.

Capacitive Level Sensor

Example 5.8

A capacitive level sensor is used to measure a liquid level. If h_{max} is 125 mm, ϵ_r is 80, and the measured capacitance change is 279.3 pF, what is the liquid level? (Given that the sensor's inner plate radius r_1 is 15 mm, the outer plate's radius r_2 is 37.5 mm.)

Outline

- Capacitors and Capacitance
- Sensing Principles and Characteristics
- Sensor Design and Applications
- **Electronic Interface Circuits**

Capacitance Output Circuits

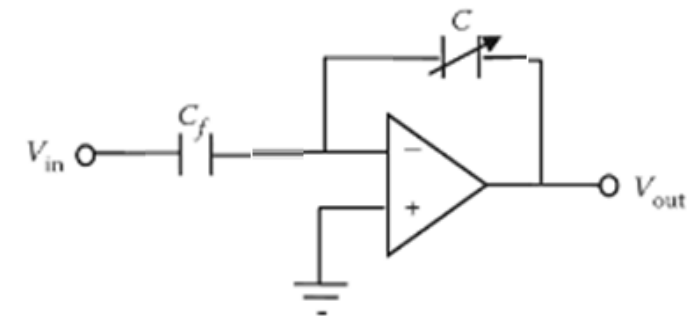
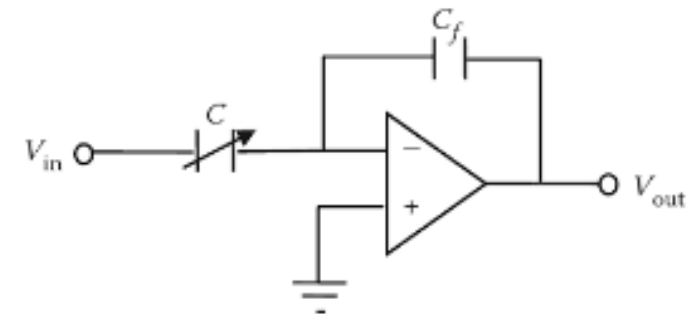
- A capacitive sensor produces a change in capacitance.
- This capacitance change can be converted into a voltage change by one of the following methods:
 - RC decay,
 - integration of current,
 - oscillator frequency,
 - AC bridge.

Amplifier

- A low-impedance amplifier circuit can be used to detect plate spacing or area.

V_{out} proportional to A $V_{out} = -\frac{C}{C_f} V_{in} = -\frac{\epsilon_0 \epsilon_r A}{C_f d} V_{in}$

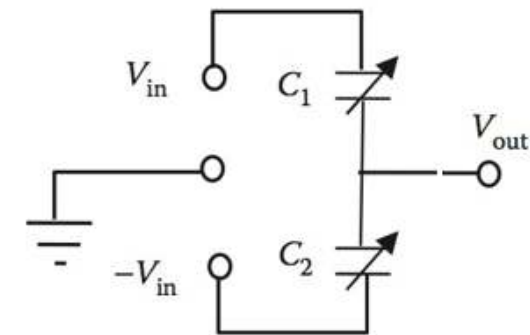
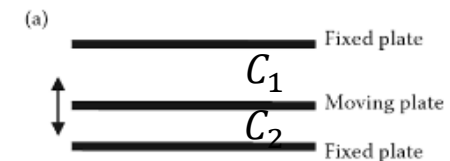
V_{out} proportional to d $V_{out} = -\frac{C_f}{C} V_{in} = -\frac{C_f d}{\epsilon_0 \epsilon_r A} V_{in}$



AC Bridge

- The capacitances between the plates C_1 and C_2 comprise a voltage divider circuit.
- Equivalent capacitance $\frac{C_1 C_2}{C_1 + C_2}$
- The output voltage V_{out} is

$$V_{out} = -V_{in} + 2V_{in} \times \frac{C_1}{C_1 + C_2}$$
$$= \frac{C_1 - C_2}{C_1 + C_2} V_{in}$$

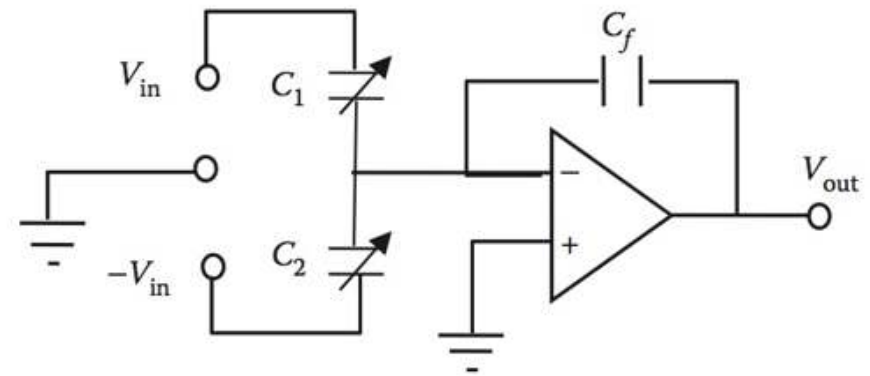


AC Bridge

- The output voltage V_{out} is

$$V_{out} = -\frac{C_1 - C_2}{C_f} V_{in}$$

where C_1 and C_2 are the capacitances between the plates.

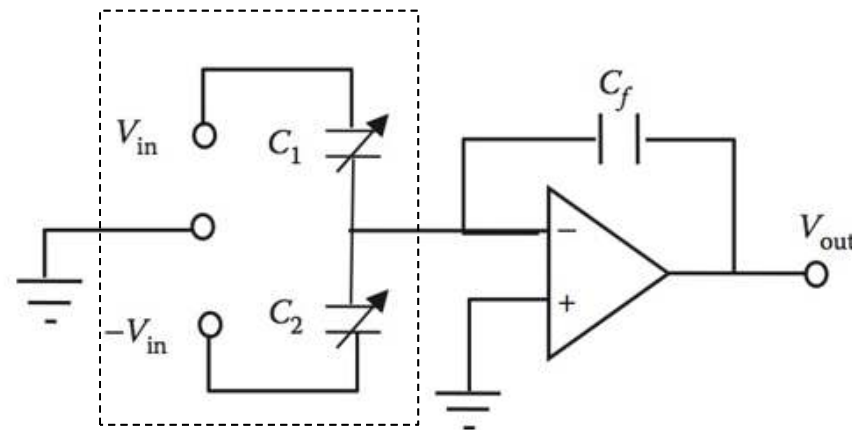


AC Bridge

Example 5.9

For the AC bridge circuit below,

- Derive the Thevenin equivalent voltage and impedance of the circuit in the dashed box as an expression of C_1 , C_2 , and V_{in} .
- Derive the expression for V_{out} as an expression of C_1 , C_2 , C_f , and V_{in} .



AC Bridge

- For the balance condition:

$$V_B = V_D$$

- Therefore,

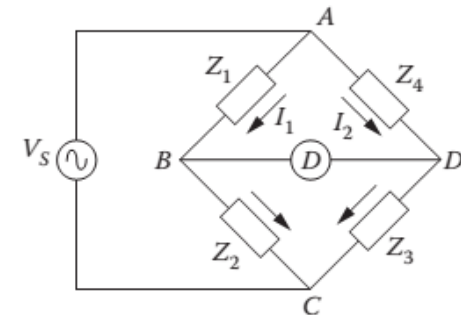
$$V_S \frac{Z_1}{Z_1 + Z_2} = V_S \frac{Z_4}{Z_3 + Z_4} \Rightarrow Z_1 Z_3 = Z_2 Z_4$$

- Since impedance is a complex number,

$$\begin{cases} \operatorname{Re}(Z_1 Z_3) = \operatorname{Re}(Z_2 Z_4) \\ \operatorname{Im}(Z_1 Z_3) = \operatorname{Im}(Z_2 Z_4) \end{cases}$$

- The complex impedance balance condition can also be expressed in polar form:

$$\begin{cases} |Z_1| |Z_3| = |Z_2| |Z_4| \\ \angle \theta_1 + \angle \theta_3 = \angle \theta_2 + \angle \theta_4 \end{cases}$$



AC Bridge

Example 5.10

An AC bridge has the following impedance: $Z_1 = 100 \Omega \angle 70^\circ$ (inductor); $Z_2 = 300 \Omega \angle -30^\circ$ (capacitor); $Z^4 = 250 \Omega \angle 0^\circ$ (resistor). Determine the unknown impedance of Z_3 .

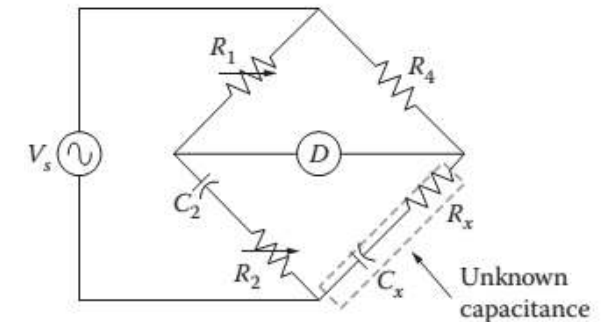
Comparison Bridge

- A comparison bridge measures an unknown capacitance or inductance by comparing it with a known capacitance or inductance.
- Under the bridge balance condition:

$$Z_1 Z_x = Z_2 Z_4 \Rightarrow R_1 \left(R_x + \frac{1}{j\omega C_x} \right) = R_4 \left(R_2 + \frac{1}{j\omega C_2} \right)$$

$$\Rightarrow \begin{cases} R_x = \frac{R_2 R_4}{R_1} \\ C_x = \frac{R_1 C_2}{R_4} \end{cases}$$

- By varying R_1 and R_2 to let the comparison bridge reach balance condition, the unknown capacitance can be derived.



- Sources:

- Winncy Y. Du, Resistive, Capacitive, Inductive, and Magnetic Sensor Technologies, CRC Press, 2014/12/09, ISBN 978-1439812440.